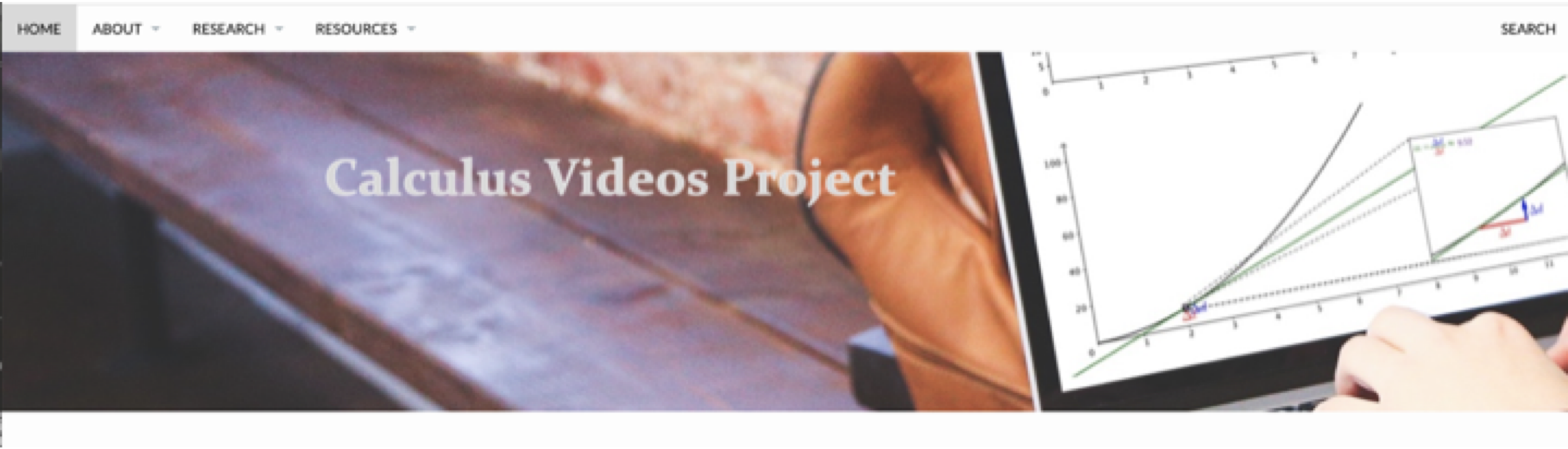
Videos Developing a Conceptual Foundation for Calculus



This material is based upon work supported by the National Science Foundation under Awards DUE # 1712312, DUE # 1711837 and DUE # 1710377

Jason Martin
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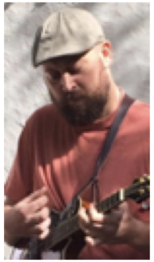
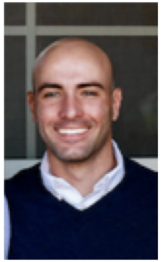
Meet the Project Team

Aaron Weinberg



Matt Thomas



Michael Tallman

Jason Martin

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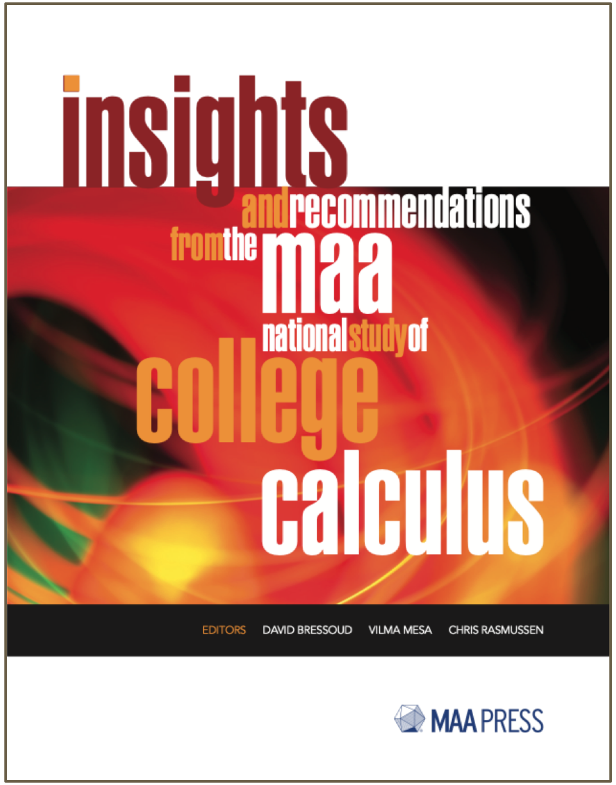
Calcvids.org

Videos Developing a Conceptual Foundation for Calculus

- A Call for Active Classrooms
- Some Design Principles for Videos in Context
- Overview of Calculus 1 Video Sequence
- A Close Look:
  - Constant Rate of Change to Derivative
  - Riemann Sums to FTOC
- Our Study (Analysis is Ongoing)

The National Situation in First Semester Calculus



MAA
MATHEMATICAL ASSOCIATION OF AMERICA

Bressoud, Mesa, & Rasmussen, (2015)

The National Situation in First Semester Calculus

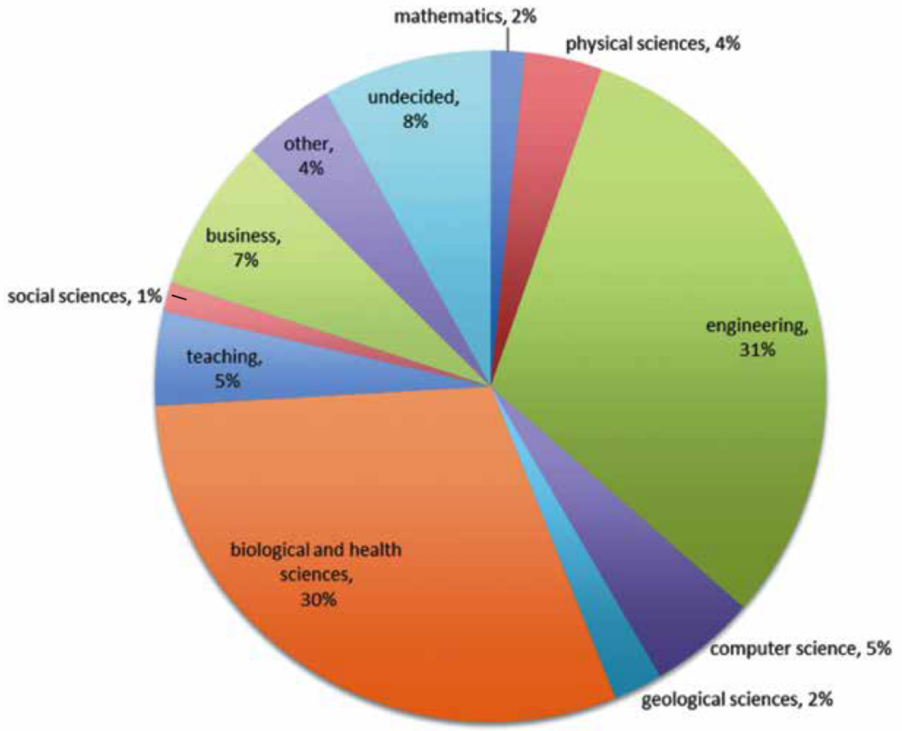
- 14,000+ students
- 213 institutions:
  - Ph.D. granting in Mathematics
  - Master's granting in Mathematics (highest degree)
  - Undergraduate (Bachelor's highest mathematics degree)
  - Two-Year Institutions
- 502 instructors

MATHEMATICAL ASSOCIATION OF AMERICA

Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

The National Situation in First Semester Calculus

- First semester calculus student career goals



Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

MATHEMATICAL ASSOCIATION OF AMERICA

The National Situation in First Semester Calculus

Student expectations

Going into Calculus 1, what percentage of students expect to earn an A? At least a B?			
34%	**43%**	**54%**	**76%**

MATHEMATICAL ASSOCIATION OF AMERICA

Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

The National Situation in First Semester Calculus

Student expectations

- 54% expected to earn an A in their Calculus course
- 93% expected to earn at least a B

MATHEMATICAL ASSOCIATION OF AMERICA

Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

The National Situation in First Semester Calculus

- Reality:

Drop / Fail (≤D) / Withdraw Rate?			
18%	**27%**	**34%**	**39%**

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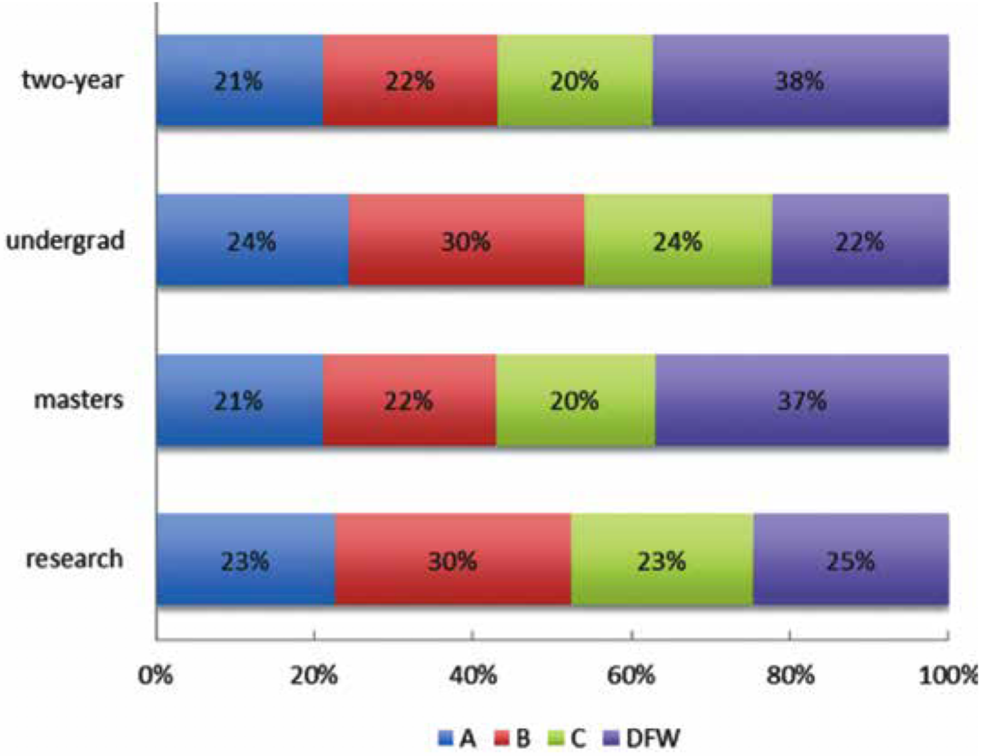
Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

The National Situation in First Semester Calculus

Reality:

- 27% overall drop-fail(≤D)-withdraw rate



Bressoud, Carlson, Mesa, & Rasmussen, (2013); Bressoud, Mesa, & Rasmussen, (2015)

MAA MATHEMATICAL ASSOCIATION OF AMERICA

Leaving STEM

- Large Institutional Study, Students Earning a C or Better:

% of students ≥C in Calc 1 & required to take Calc 2 but not persisting?			
10%	20%	30%	40%

Thompson et al., (2007)

Leaving STEM

- Large Institutional Study, Students Earning a C or Better:
  - **33%** in Calc 1 with major requiring Calc 2 did not persist in Calc 2.

Thompson et al., (2007)

Leaving STEM

- Large Institutional Study, Students Earning a C or Better:
  - **33%** in Calc 1 with major requiring Calc 2 did not persist in Calc 2.
- Students were voluntarily **leaving STEM** because they were **unsatisfied with classroom culture**.
- "Most students are **not engaged by lecture** format"

Bressoud, (2012); Seymour & Hewitt, (1997); Thompson et al., (2007)

The Appeal

- Adopt **student-centered active learning** pedagogies to support student retention and success.
- Professional mathematics associations recommending the **adoption of active-learning** strategies in all math classrooms.
- STEM disciplines recommending more problem-solving and **in-class activities** with small groups.

Braun et al., (2016); Bressoud, Mesa, & Rasmussen, (2015); Freeman et. al, (2014); NAE, (2005); NRC, (2011); & PCAST (2012)

Enter: The Calculus Videos Project (CVP)

- Video lessons provide an opportunity for instructors to create more student-centered active classrooms.
  - Save instructional time
  - Support flipped and blended instruction
  - Students are already using videos (e.g. Khan Academy, YouTube)
- Videos lessons can provide students the opportunity to engage with the dynamic nature of calculus
- COVID changes things… videos are essential for the online class environment.

Informing Video Design: Resources on CalcVids.org

calcvids.org/instructorinfo/

HOME ABOUT RESEARCH RESOURCES

Information about the Concepts and Videos

- A flowchart that shows the core videos and the relationships between the concepts in the videos
- A description of the key concepts, terminology, and notation used in the videos.
- A description of the theory of quantitative reasoning, which is a central component of our video design and course structure
- A description of how quantitative reasoning is used in calculus
- Additional information about quantitative reasoning:
  - A description of the concept of quantitative reasoning by Michael Tallman
  - A paper by Pat Thompson that describes all of the technical details of quantitative reasoning
  - A paper by Moore, Carlson, and Oehrtman that provides an example of using quantitative reasoning to describe students' thinking about precalculus problems
- A description of intellectual need-provoking tasks that we have created to support instruction

Instructional Resources

- Suggestions for Incorporating Videos into Instruction
- Versions of the materials that can be directly imported into various learning management systems (e.g., Canvas, Blackboard, etc.)
- Powerpoint Files you can use to create your own versions of the videos
- Additional homework problems you can assign to your students
- Intellectual Need-Provoking Tasks you can use for in-class problem-solving and discussion

Support

- You can contact any member of the research team at any time:
  - For most questions, contact Aaron Weinberg (aweinberg@ithaca.edu)
  - For questions about quantitative reasoning or other mathematical/learning theory, contact Michael Tallman (michael.tallman@okstate.edu)
  - For questions about the Geogebra animations we use, contact Jason Martin (jasonm@uca.edu)
  - For questions about Ximera, contact Matt Thomas (mthomas7@ithaca.edu)

Informing Video Design

Intellectual Need

Quantitative Reasoning

Necessity Principle: Students are most likely to learn when they see a need for what we intend to teach them, where by "need" is meant intellectual need, as opposed to social or economic need. -Guershon Harel (1998)

Intellectual Need

An internal drive experienced by a learner to solve a problem

Composition	Position along path (km)	Amount of dust per distance traveled (mg/km)
Very sandy	0	6
Moderately sandy	20	3.5
Slightly sandy	40	2.5
Slightly rocky	60	2
Moderately rocky	80	1.5
Very rocky	100	1

Create disequilibrium, students feel need to…

- Compute
- Be efficient
- Organize ideas
- Find structure
- Understand
- Explain
- Communicate

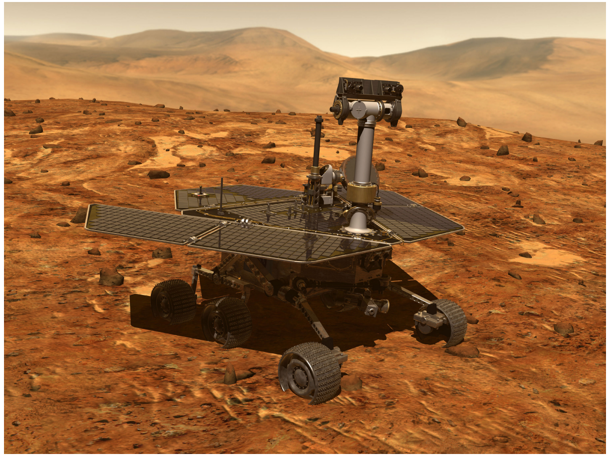
Informing Video Design

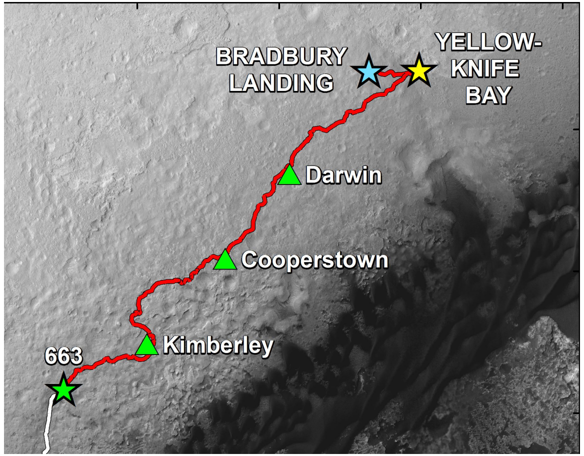
Intellectual Need

Quantitative Reasoning

A Thought Experiment: The Rover Problem

The Opportunity rover (pictured below) landed on Mars in 2004 and has been actively exploring the planet ever since. It is powered by solar cells. As the rover travels across the Martian surface, it kicks up dust, which accumulate on its solar cells. The amount of dust that it kicks up depended on the composition of the surface it was traveling over—a rockier surface kicks up less dust than a softer surface. When planning a path for the rover to follow, scientists need to know how far it might travel before too much dust accumulates on its solar panels.





What does a student first encountering this problem, need to know?

A Thought Experiment: The Rover Problem

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What if this table was provided?

Composition	Position along path (km)	Amount of dust per distance traveled (mg/km)
Very sandy	0	1
Moderately sandy	20	1.5
Slightly sandy	40	2
Slightly rocky	60	2.5
Moderately rocky	80	3.5
Very rocky	100	6

What does a student first encountering this problem, need to know?

A Thought Experiment: The Rover Problem

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What if this formula was provided?

$$R(p) = \frac{6}{\sqrt{\frac{p}{50} + 1}} \quad \text{mg/km}$$

What does a student first encountering this problem, need to know?

Informing Video Design: Quantitative Reasoning

To model mathematical phenomena, a student must first conceive of relevant measurable attributes (**quantities**) of the phenomena, relate these attributes to one another, and operate on these attributes (**quantitative reasoning**).

Smith & Thompson, (2007); Thompson, (1990, 2011)

Informing Video Design: Quantitative Reasoning

Quantitative reasoning is a characterization of the mental actions involved in conceptualizing situations in terms of quantities and quantitative relationships.

Smith & Thompson, (2007); Thompson, (1990, 2011)

Informing Video Design: Quantitative Reasoning

Covariational reasoning refers to the mental actions involved in coordinating the values of two varying quantities while attending to how these values change in relation to each other (Carlson et al., 2002).

Carlson et al., 2002; Saldanha & Thompson, 1998; Thompson & Carlson, 2017

Informing Video Design: Quantitative Reasoning

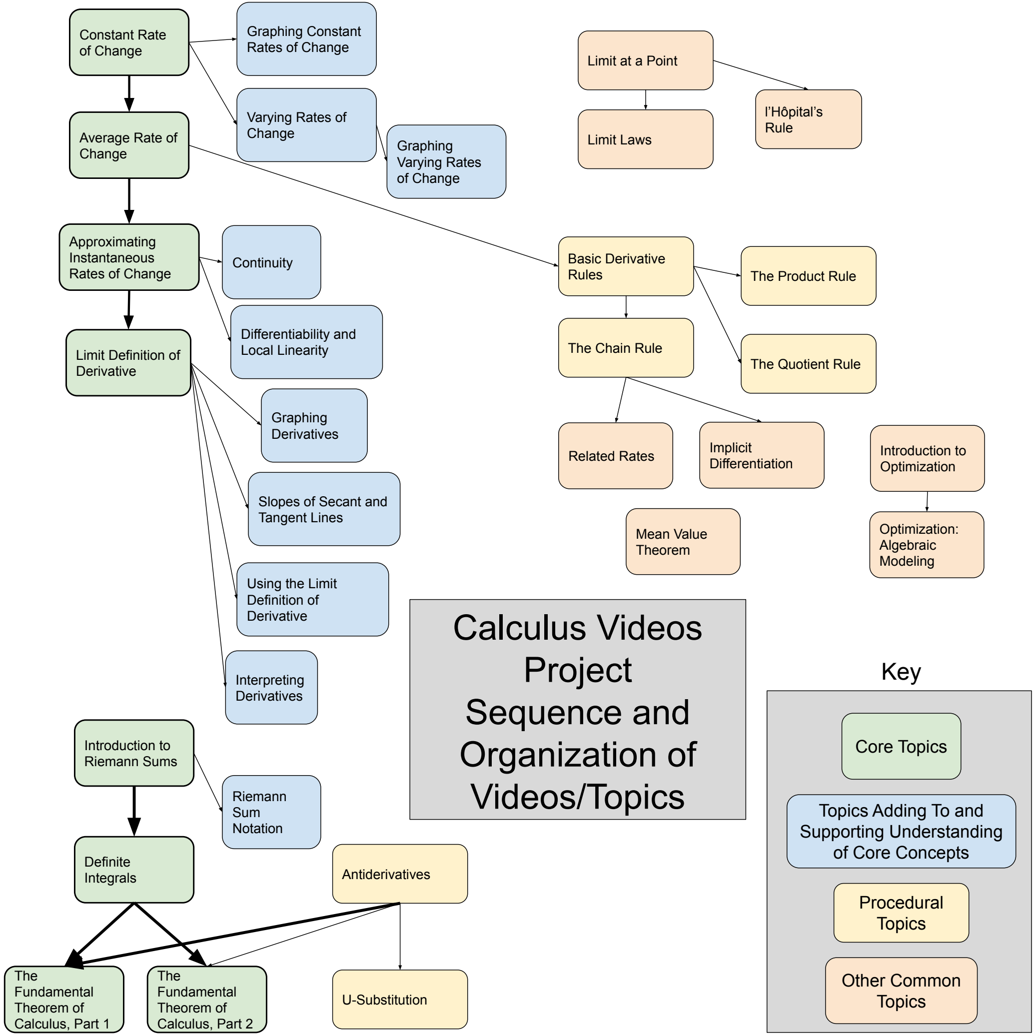
Our contextual videos

- Prompt students to identify and coordinate measurable attributes (i.e., quantities) as they covary
- Represent the relationship between covarying quantities symbolically, graphically, and numerically
- Attend to frames of reference and units of measure
- Emphasize quantitative (as opposed to arithmetic) operations

Our Videos

- ~30 video sets over different Calculus 1 topics
    - 1-3 videos per set
- Over 50 videos
- Video Types:
    - Conceptual
    - Procedural
    - Need Inducing videos

Our Videos


Constant Rate of Change
Graphing Constant Rates of Change
Varying Rates of Change
Graphing Varying Rates of Change
Average Rate of Change
Approximating Instantaneous Rates of Change
Continuity
Differentiability and Local Linearity
Limit Definition of Derivative
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Slopes of Secant and Tangent Lines
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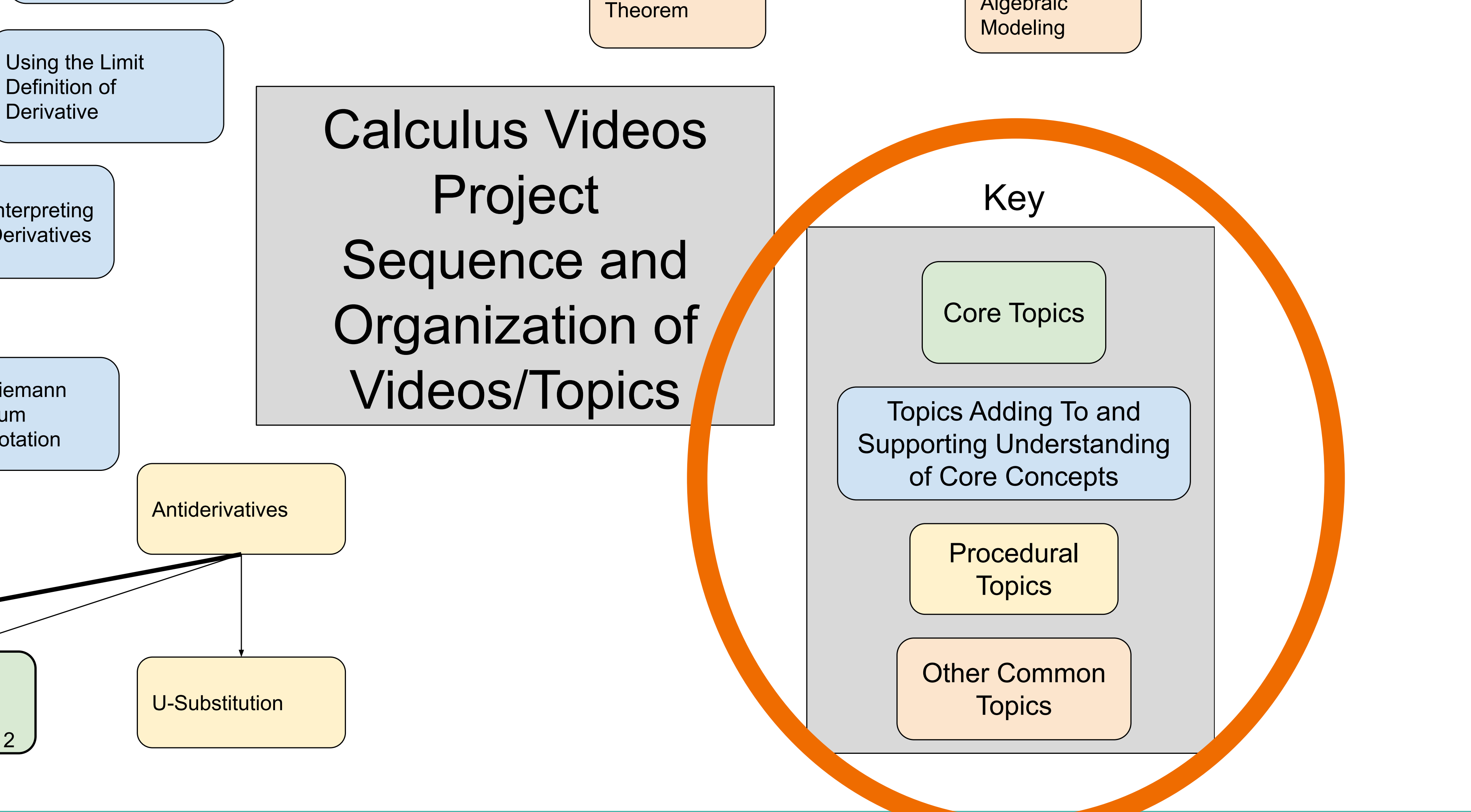
U-Substitution
Limit at a Point
Limit Laws
l'Hôpital's Rule
Basic Derivative Rules
The Product Rule
The Chain Rule
The Quotient Rule
Related Rates
Implicit Differentiation
Introduction to Optimization
Optimization: Algebraic Modeling
Mean Value Theorem
Calculus Videos Project Sequence and Organization of Videos/Topics
Key
Core Topics
Topics Adding To and Supporting Understanding of Core Concepts
Procedural Topics
Other Common Topics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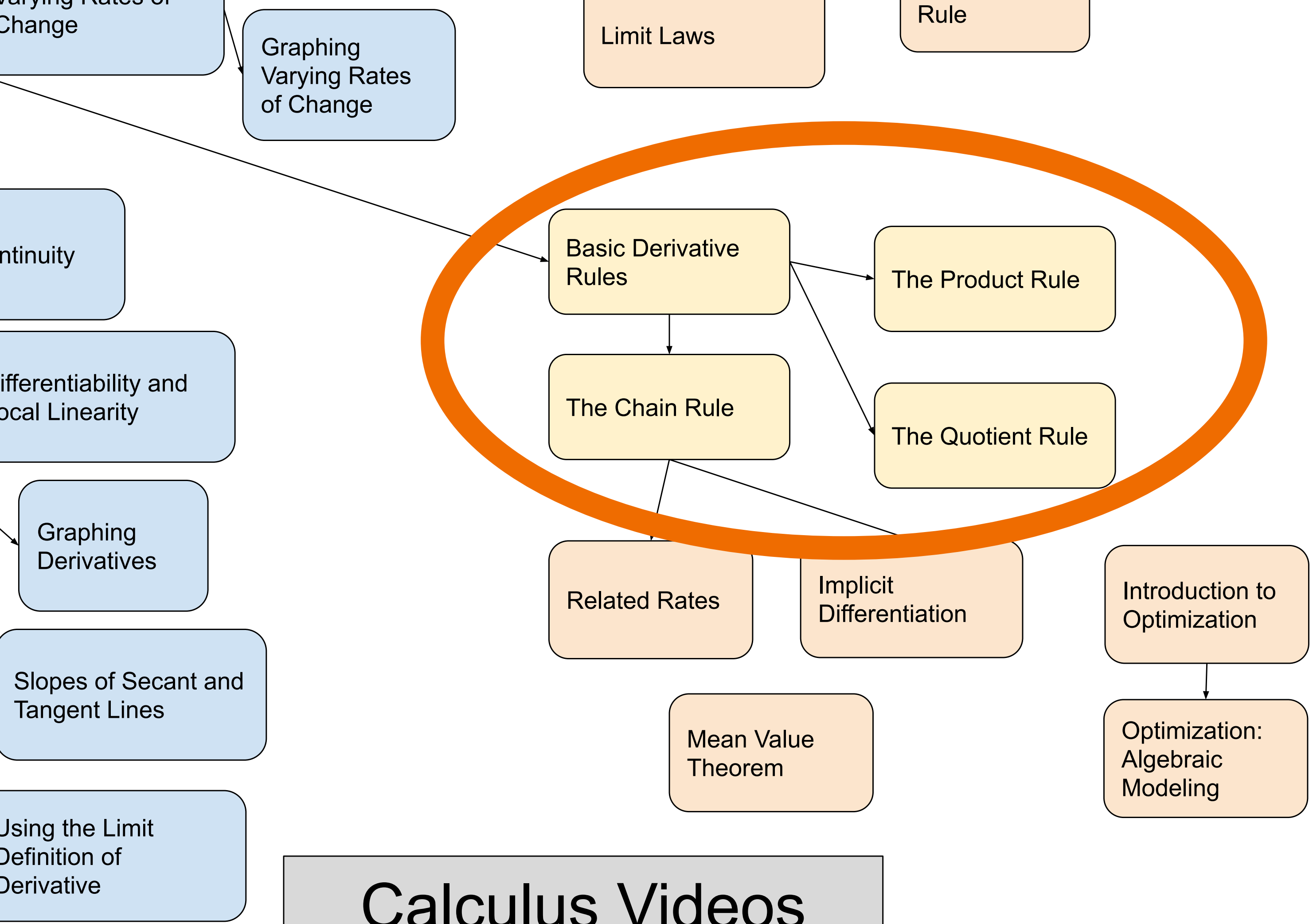
2



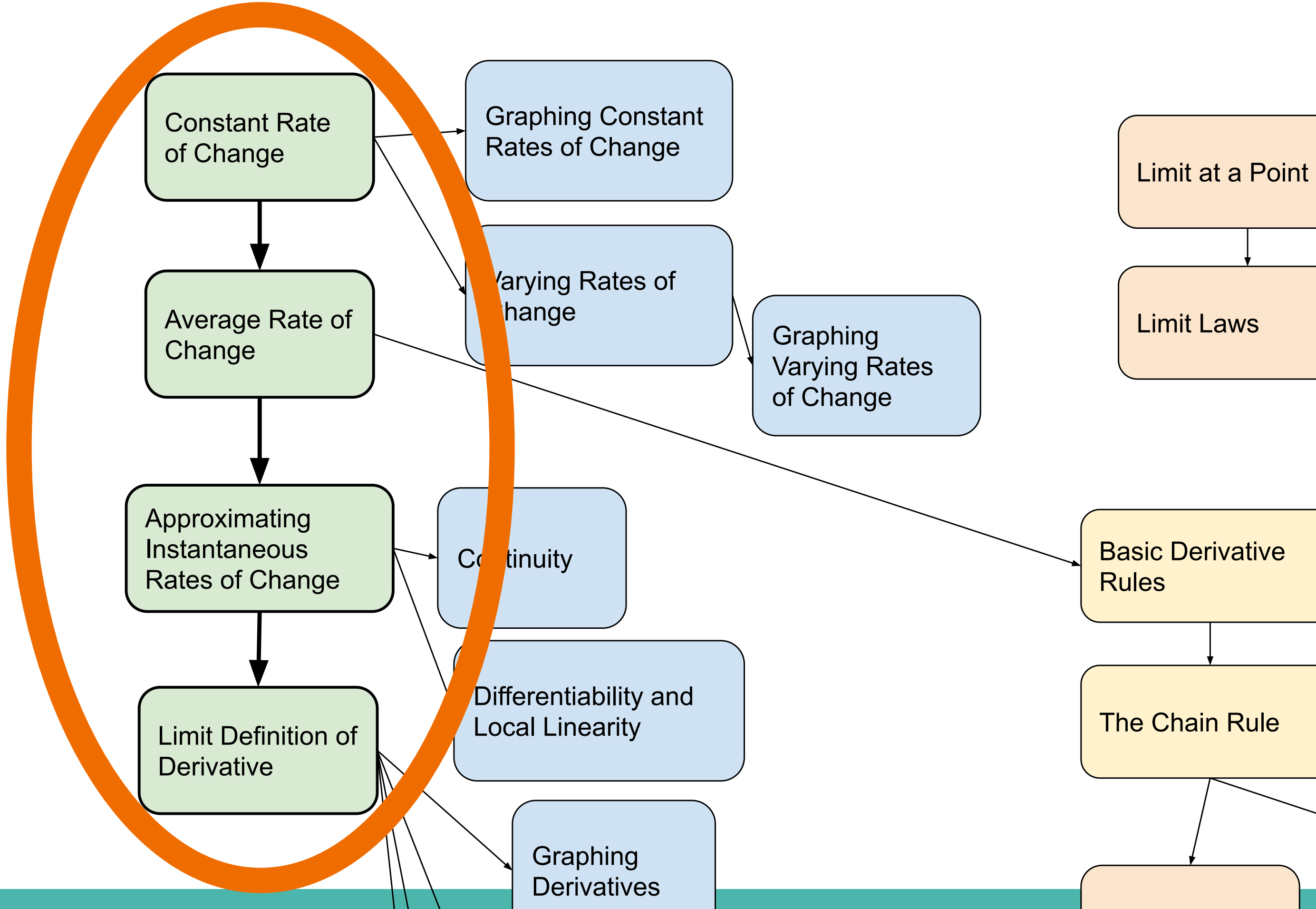
Calculus Videos

Procedural Videos: Sample Video (Product Rule)

Product of Two Functions

$$h(x) = \underbrace{(3x - 2)}_{f(x)}\underbrace{(2x + 1)}_{g(x)} = 6x^2 - x - 2$$

Our Videos



Constant Rate of Change
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Continuity
Differentiability and Local Linearity
Graphing Derivatives
Limit at a Point
Limit Laws
Basic Derivative Rules
The Chain Rule

Video Sets: CROC to Limit Def. of the Derivative

- Constant Rate of Change (CROC) & Graphing CROC
- Varying Rates of Change & Graphing Varying Rates
- Average Rate of Change (AROC):
  Define AROC in terms of CROC
- Approximating Instantaneous Rates of Change:
  Using AROC over small interval
- Limit Definition of Derivative:
  Limit over shrinking interval to connect CROC with IROC

A Conceptual Development of Constant Rate

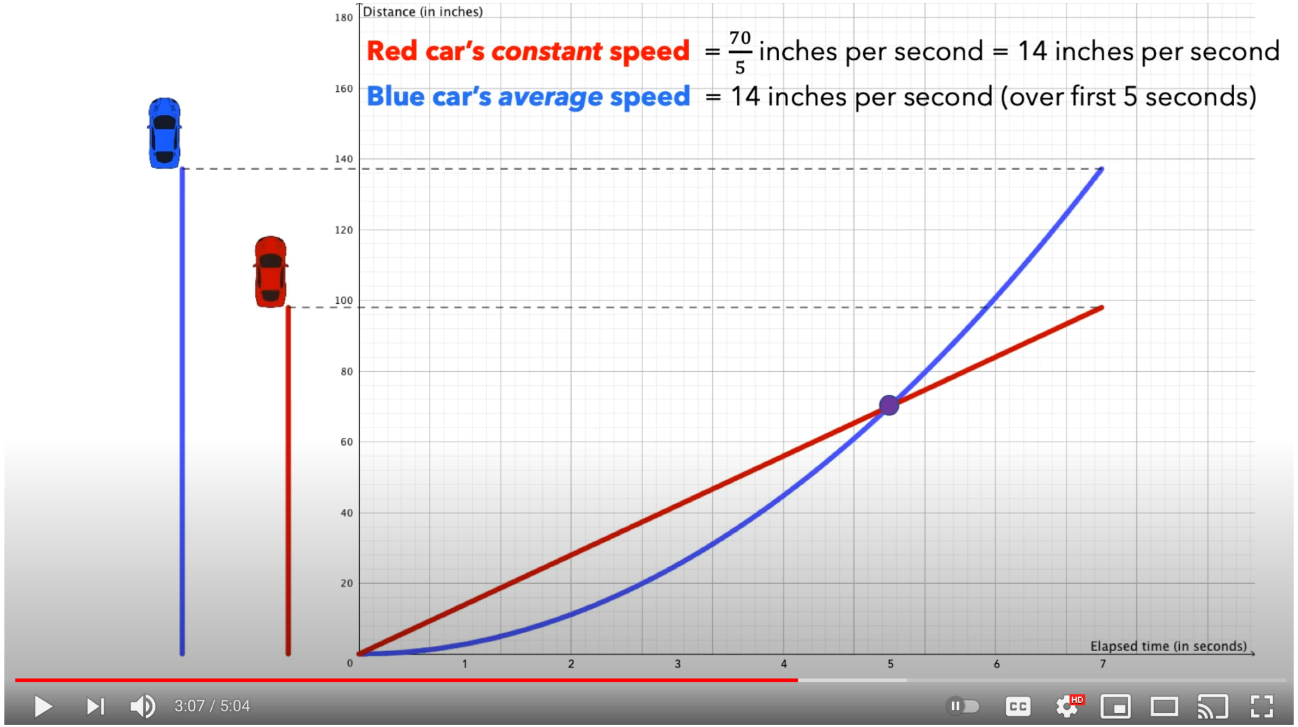
- First: A student must conceive the continuous variation of each quantity.
- Rate of change is constant if changes in quantities measure are proportional.
- Quantities' measures require attention to point of reference.
- If the variance of f with respect to x is the constant rate m, then the amount of change of f is m times as much as the amount of change in x.

Pouring Water

Rates of Change

Constant Rate to Average Rate

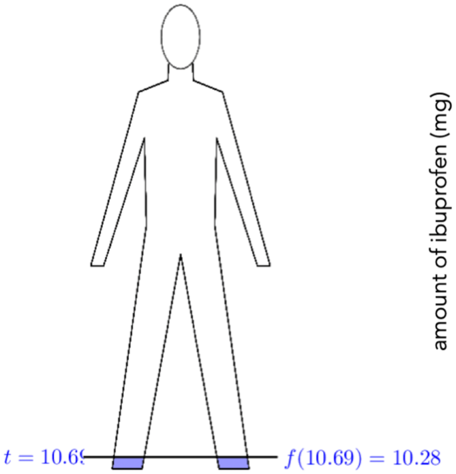
- Ave Rate is a constant rate… within an imagined situation/object to cover the same change in the dependent quantity over the same variation of the independent quantity

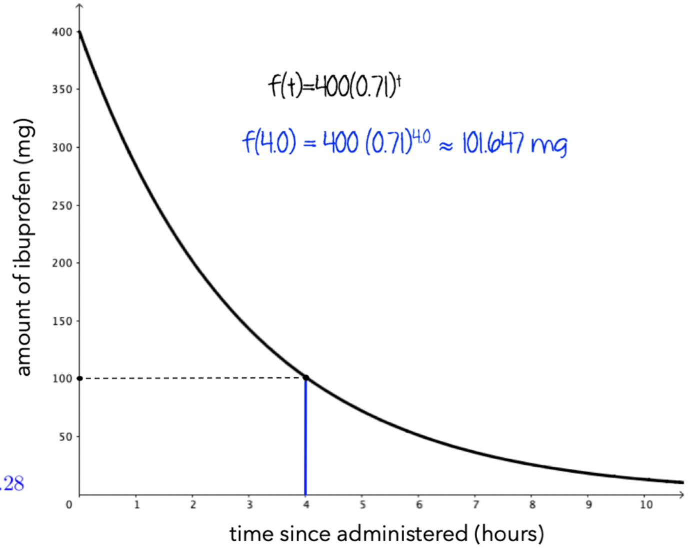


A Conceptual Development of Derivative

- Leverage constant rate of change.
- Derivative as limit of average rates.
- For intervals small enough, a function varies at essentially a constant rate.
- Over small enough intervals, the amount of change in f is $f'(a)$ times as much as as the change in x.

How quickly is ibuprofen leaving the body 4 hours after being administered?





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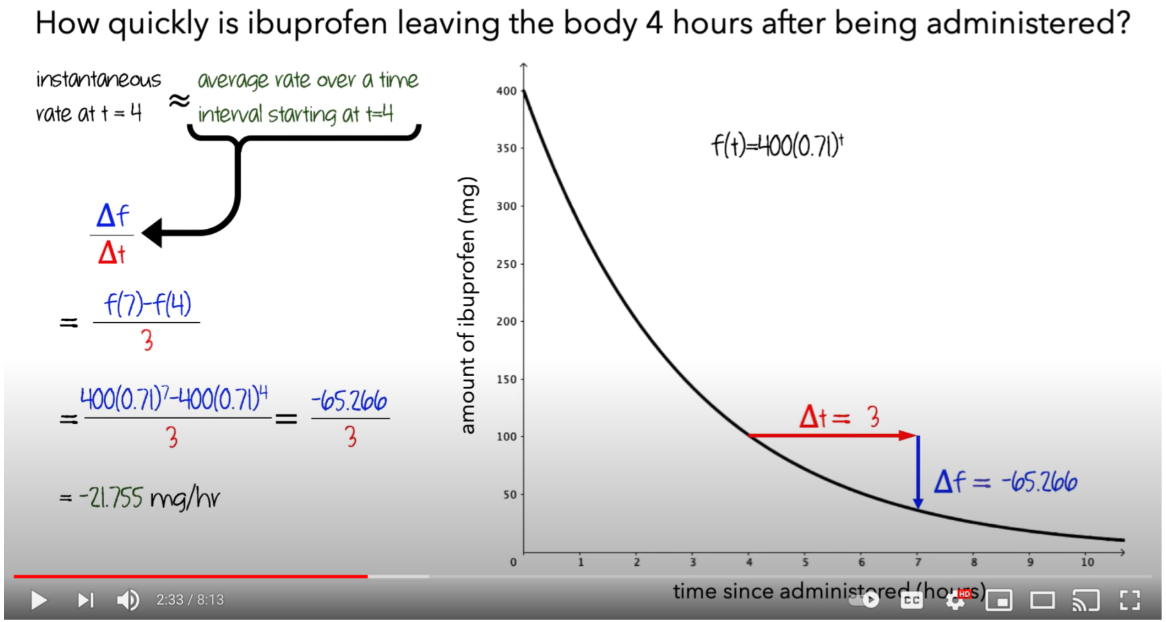

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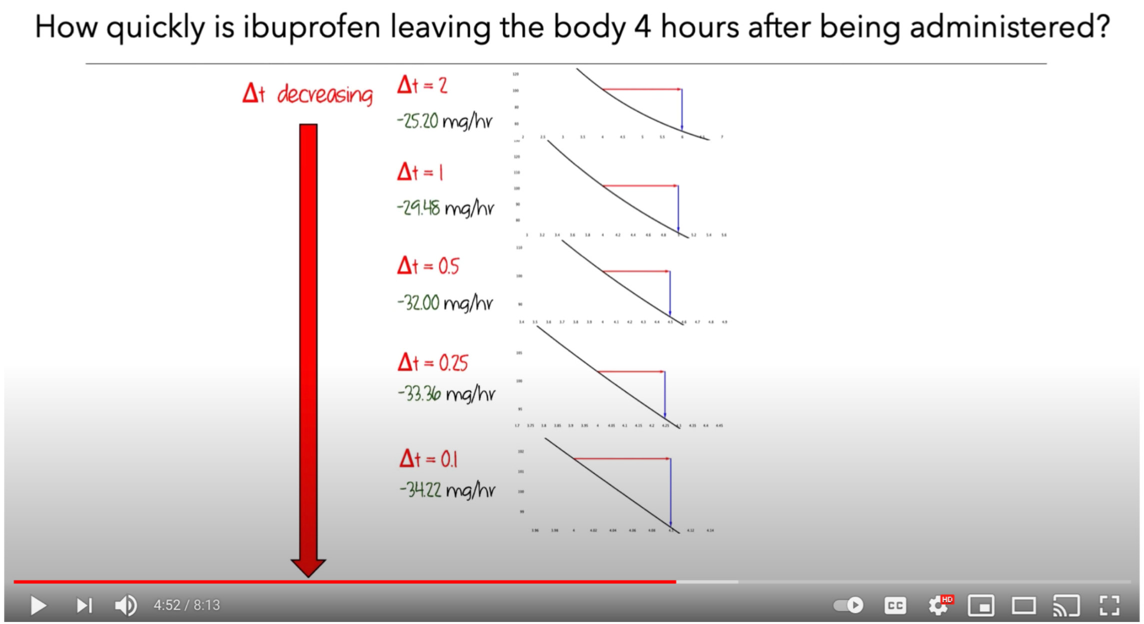

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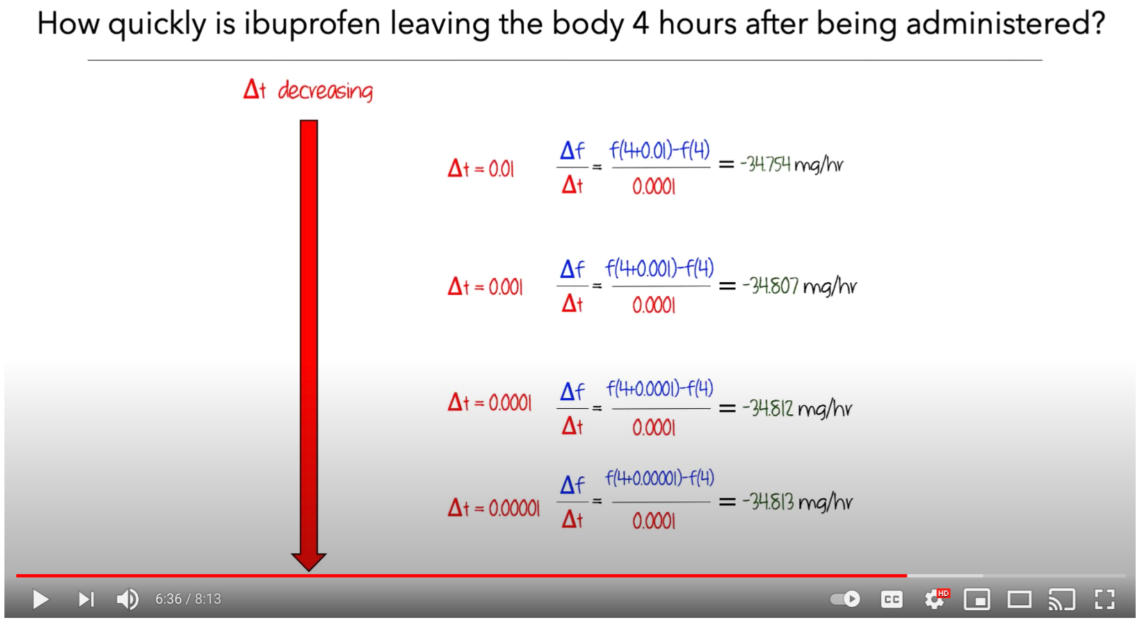

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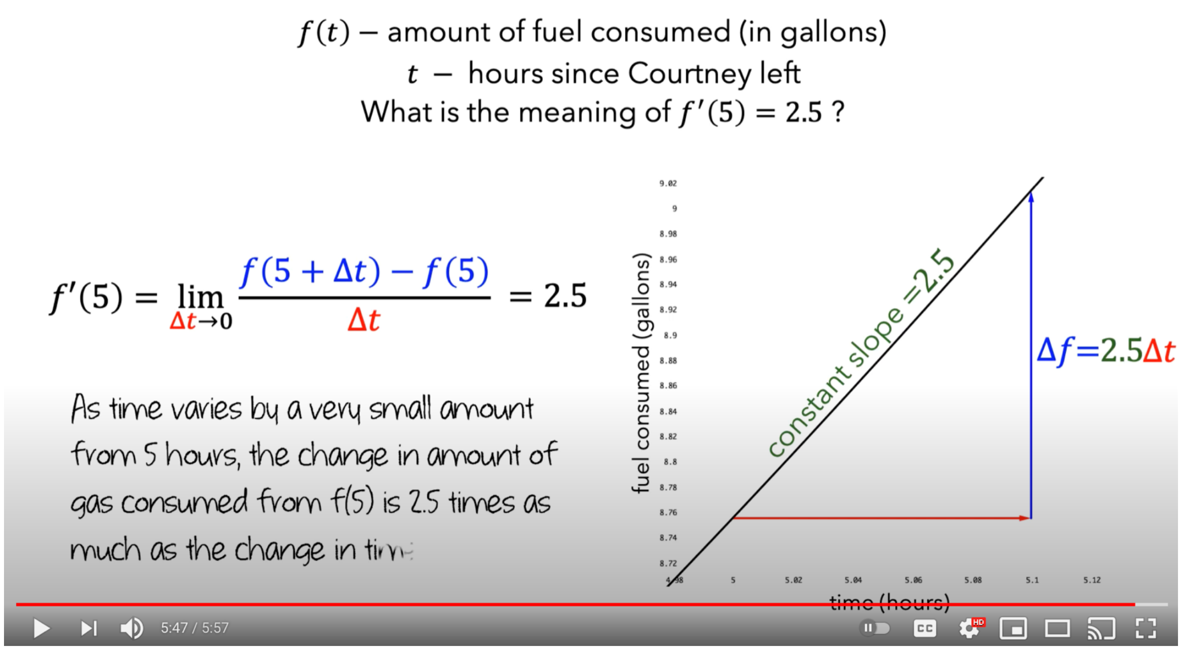

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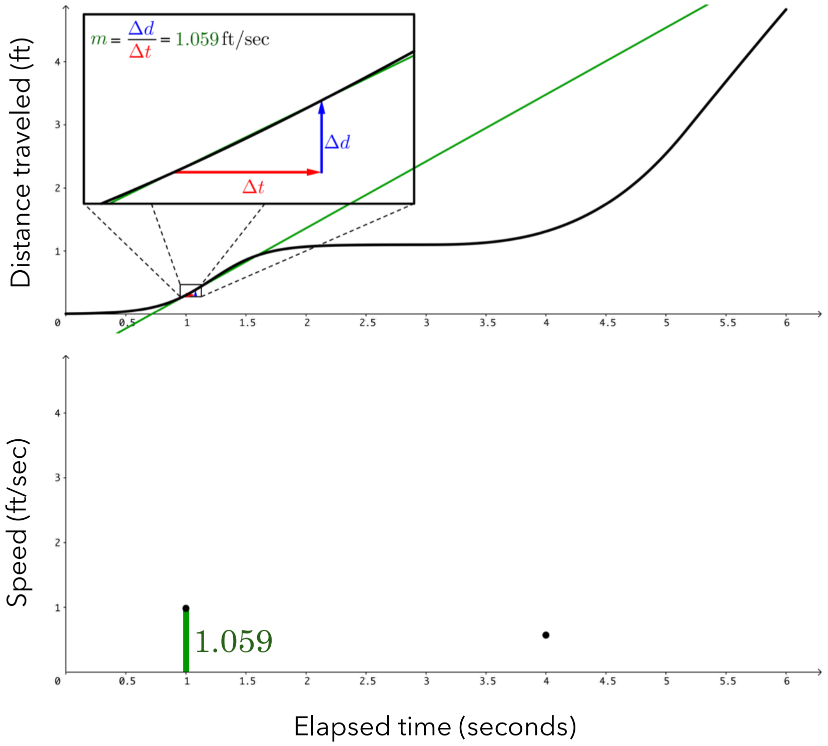


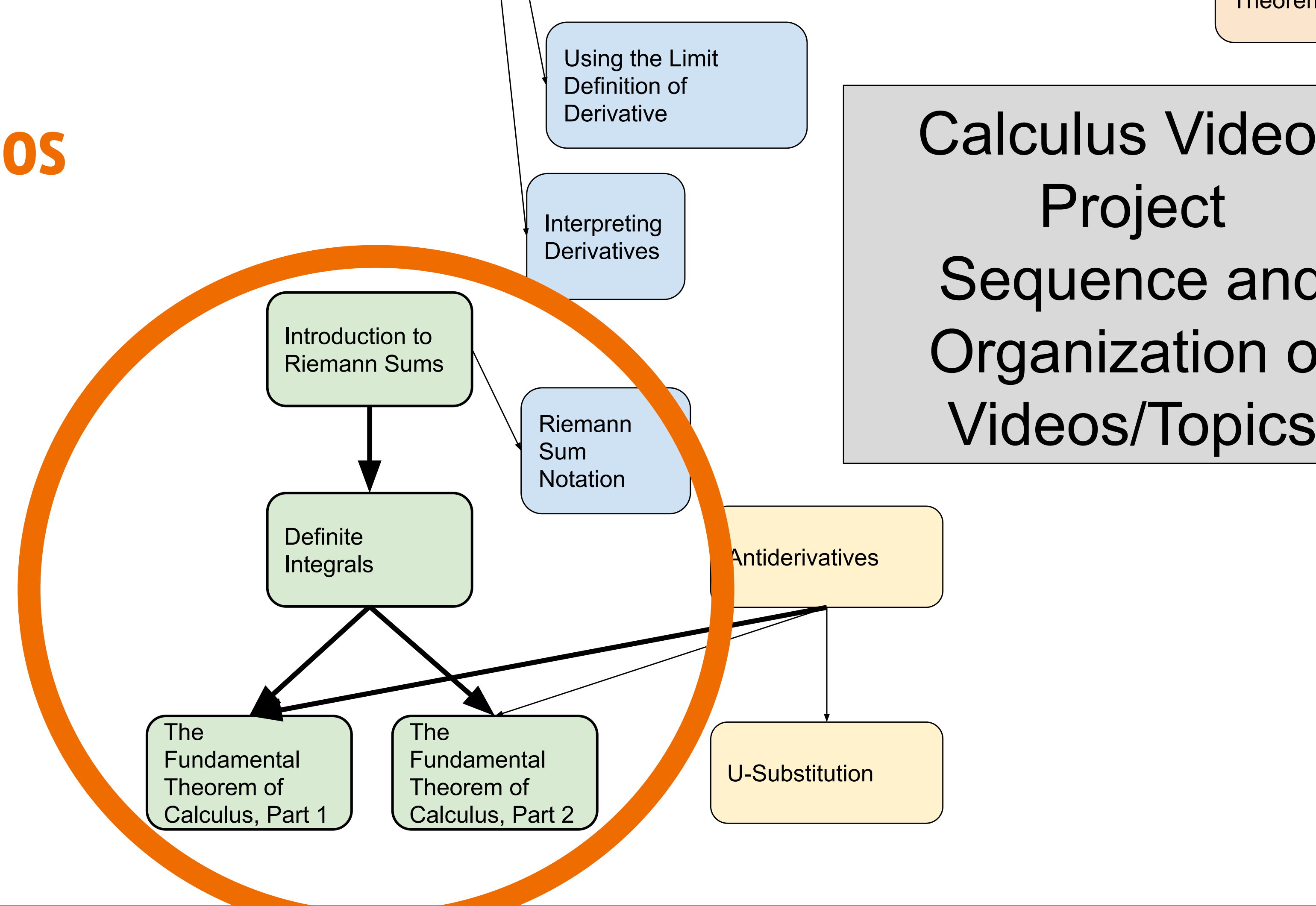
Constant Rate of Change as a Foundation for Ideas

- A mature understanding of rate relies upon developed images of continuous variation that include two quantities and how they covary.
- Quantity: Measurable attribute of an object
  - Speed: Distance & Time
  - Container: Volume and Height
  - Etc.
- Rate: Defines a proportional relationship between varying quantities' measures.

Constant Rate of Change as a Foundation for Ideas

- Instantaneous rates of change (derivatives) are defined as limit of average rates of change.
- An average rate of change of a function over an interval is the constant rate of change of a linear function (slope of secant line) over the same interval.
- When zooming to illustrate derivatives, we observe that for intervals small enough, a differentiable function varies at essentially a constant rate (linear).
- Riemann sums in context includes imagining a varying rate as if it is constant over successive small intervals.

Our Videos



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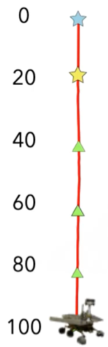
Riemann Sums to FTOC Conceptual Development

- Computing accumulation when rate is non-constant
- Riemann Sums
  - Are *not* (defined as) “area under the curve”
  - Are approximations of accumulation of a quantity over an interval of another quantities’ variation
- Definite integrals are exact accumulation of quantities

Riemann Sums to Fundamental Theorem

- Begin by assuming rate is constant over successive uniform intervals of the independent quantity's variation.



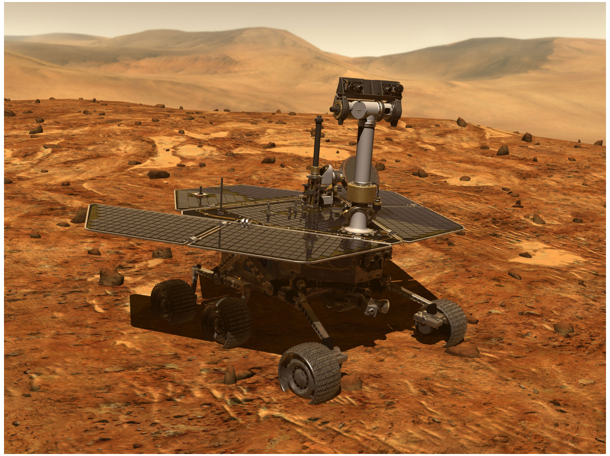
Composition	Position along path (km)	Length of Interval	Amount of dust per distance traveled (mg/km)	Accumulated Dust
Very sandy	0	20 Km	6	120 mg
Moderately sandy	20	20 Km	3.5	70 mg
Slightly sandy	40	20 Km	2.5	50 mg
Slightly rocky	60	20 Km	2	40 mg
Moderately rocky	80	20 Km	1.5	30 mg
Very rocky	100		1	

310 mg

Approximation of total amount of dust

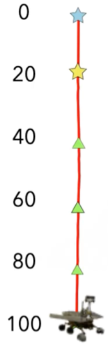
Assume each rate is constant over an interval

6:01 / 6:49 Approximate the total amount of dust that accumulates



Riemann Sums to Fundamental Theorem

- Calculated approximations to the accumulations each intervals of the independent quantity's variation and add.



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310 mg

Approximation of total amount of dust

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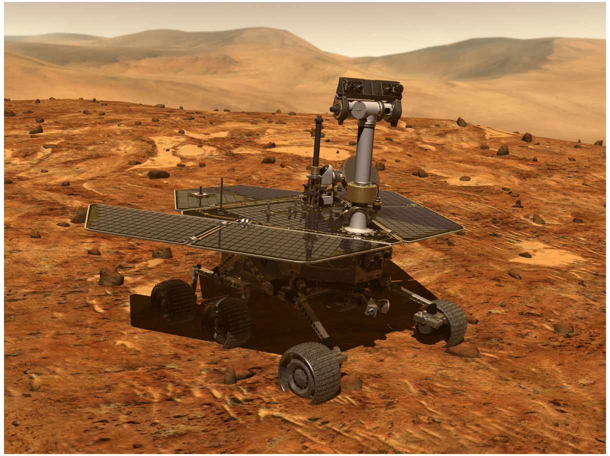


Finding Total Accumulation

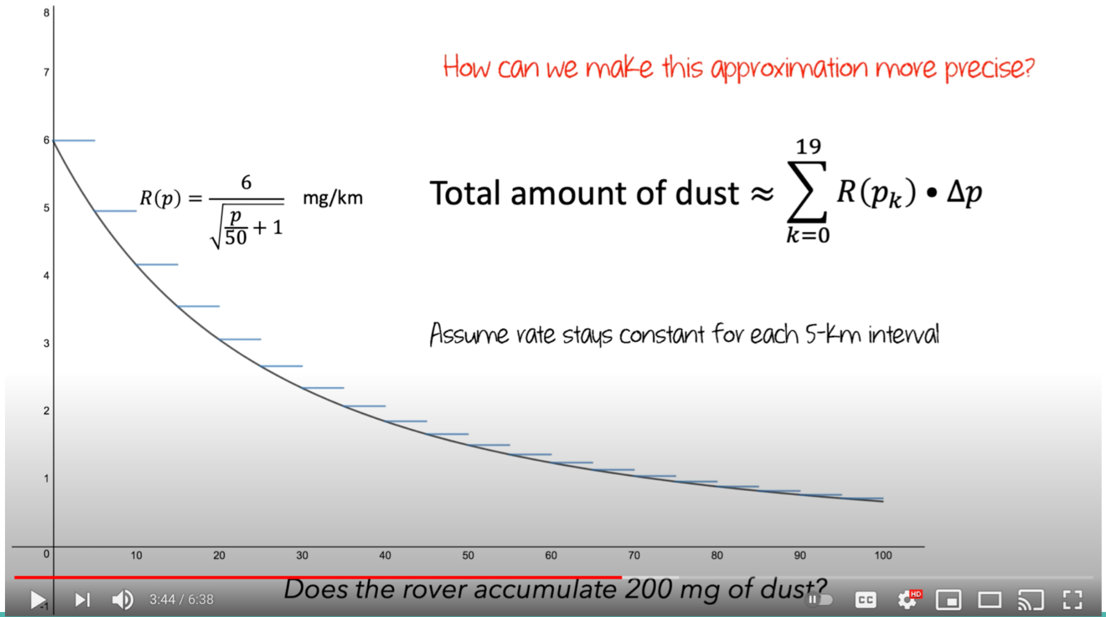
Treat each interval as having a constant rate of dust accumulation

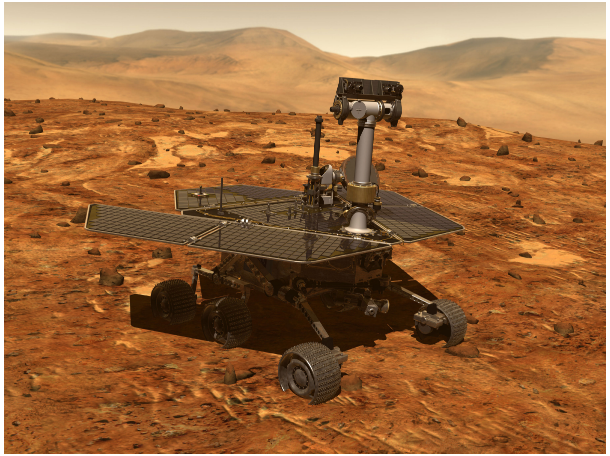
Accumulation for each interval = Constant rate x Change in position

Add up amounts from each interval to approximate total accumulation

Riemann Sums to Fundamental Theorem

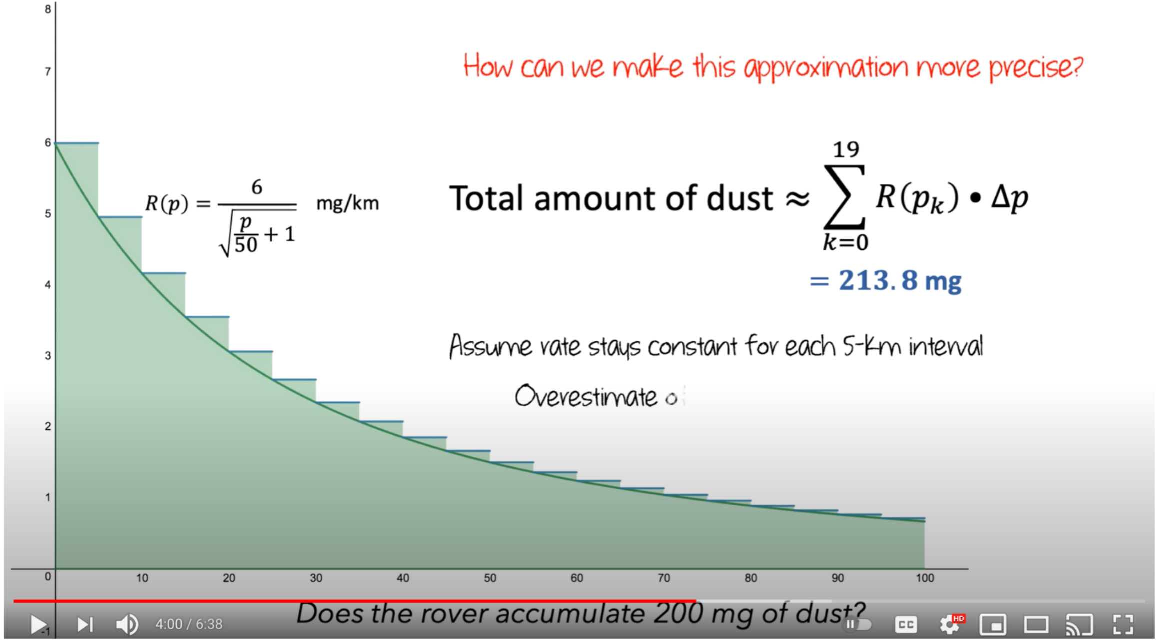
- Graph an imagined rover where rate is constant over successive uniform intervals of the independent quantity's variation.

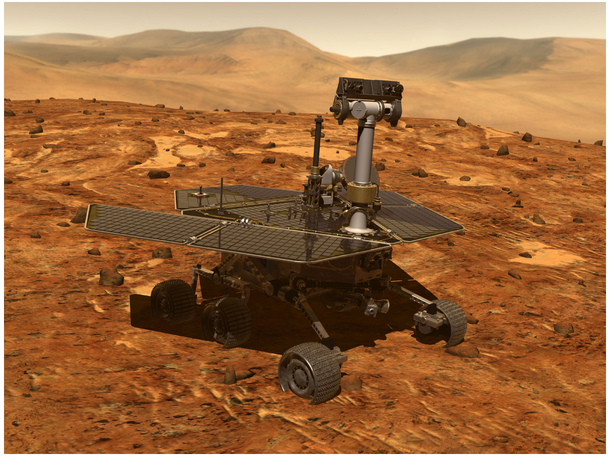




Riemann Sums to Fundamental Theorem

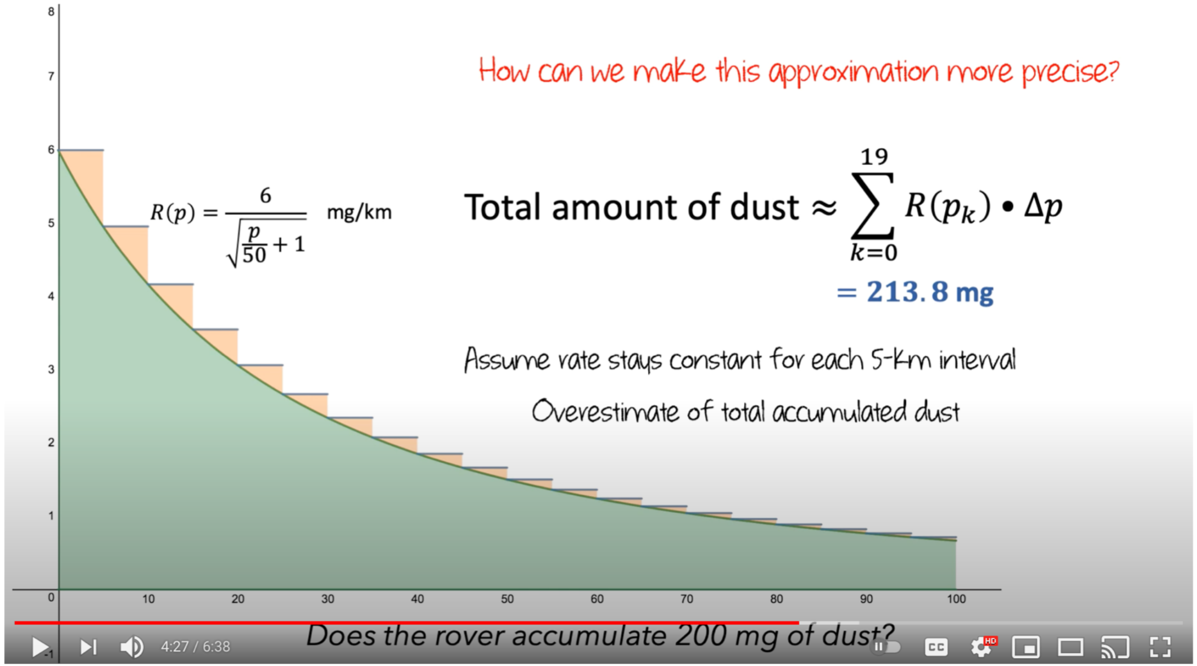
- Approximations are made more precise by including more divisions of interval over which the independent quantity varies.

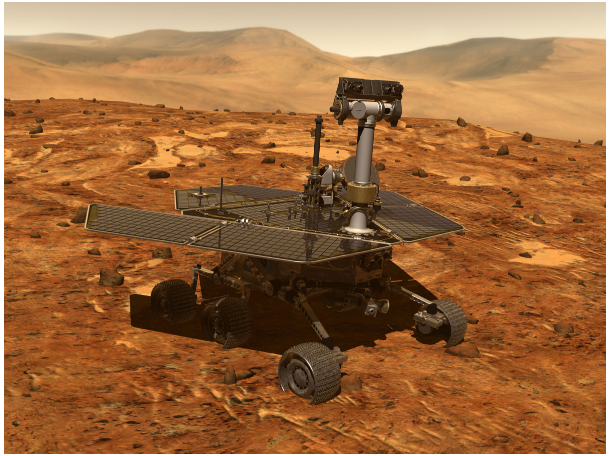




Riemann Sums to Fundamental Theorem

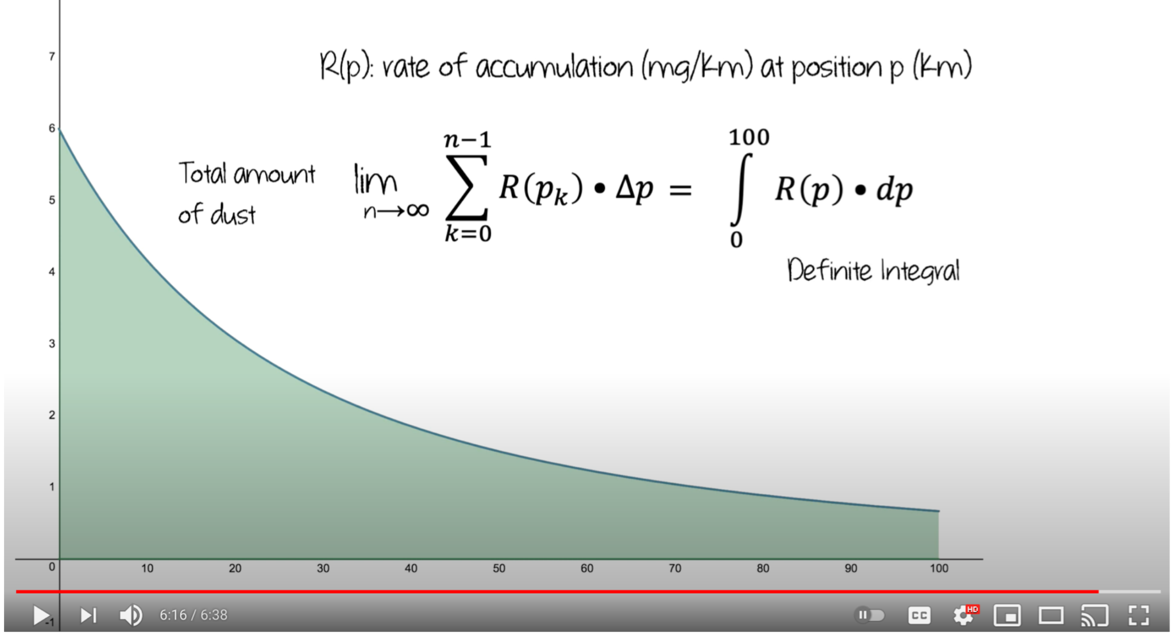
- With corresponding error.

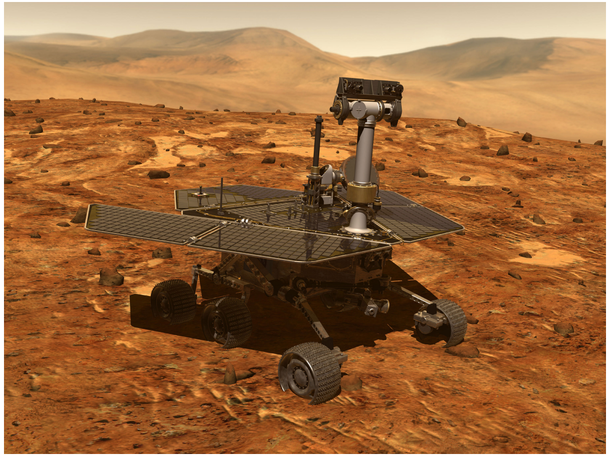




Riemann Sums to Fundamental Theorem

- Made exact through the limit.
- **The definite integral emerges as a model of the total accumulation.**

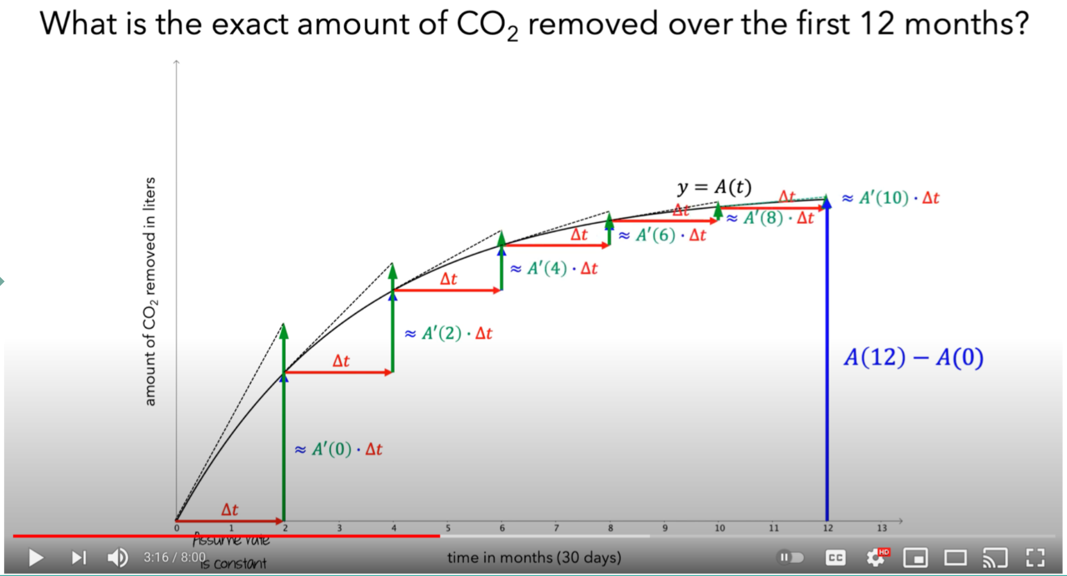




to Fundamental Theorem

- Leverage constant rate of change over these intervals
  - $\Delta f \approx f'(a)\, \Delta x.$
  - Total accumulation is approximated by the sum of accumulations over these intervals.

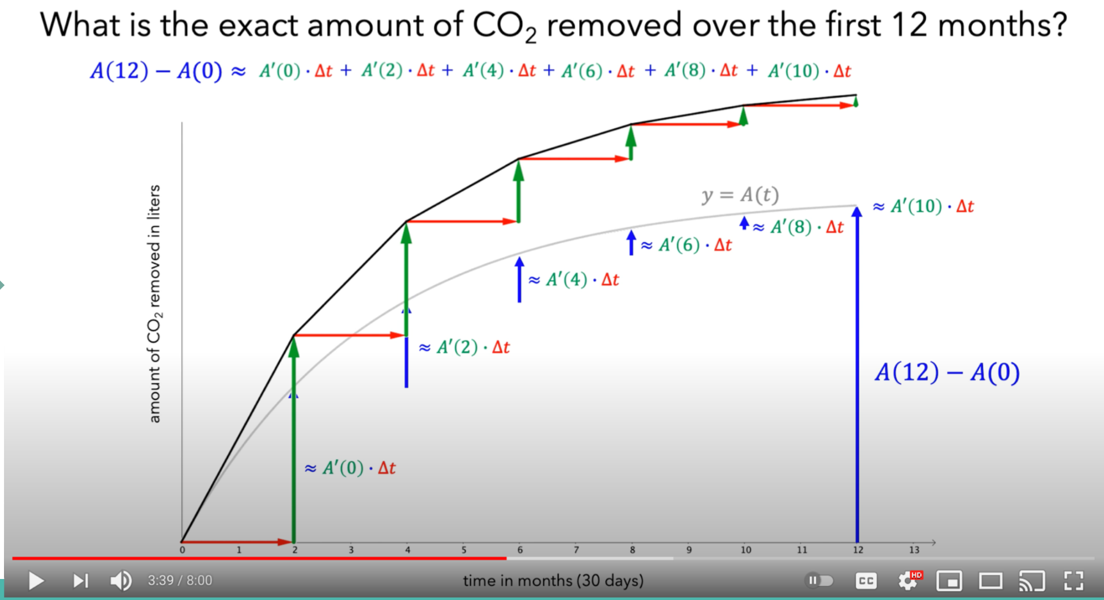
Linear Approximations over successive intervals



to Fundamental Theorem

- Leverage constant rate of change over these intervals
  - $\Delta f \approx f'(a)\, \Delta x$.
  - Total accumulation is approximated by the sum of accumulations over these intervals.

Cumulative Affect of Exclusively Using Linear Approximation Over the Entire Variation of the Independent Quantity

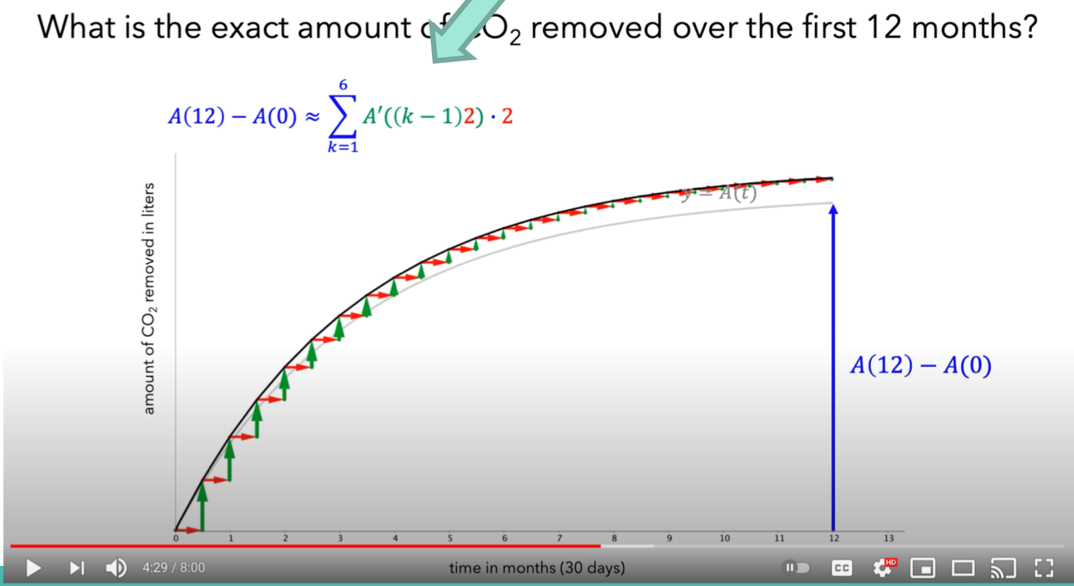


to Fundamental Theorem

- Leverage constant rate of change over these intervals
  - $\Delta f \approx f'(a)\,\Delta x$.
  - Total accumulation is approximated by the sum of accumulations over these intervals.

Riemann Sum of the Rate Function, *A'*

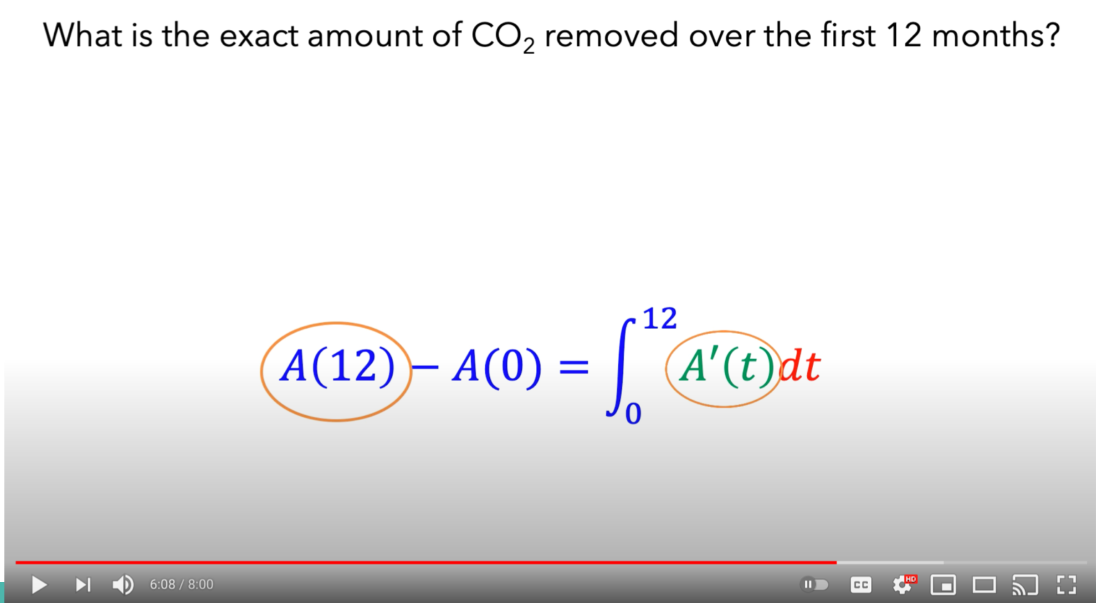
Linear Approximations Made More Precise by Using Smaller Intervals



to Fundamental Theorem

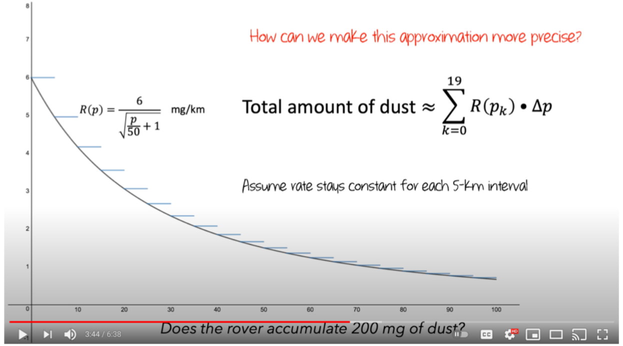
- Leverage constant rate of change over these intervals
  - $\Delta f \approx f'(a)\, \Delta x.$
  - Total accumulation is approximated by the sum of accumulations over these intervals.

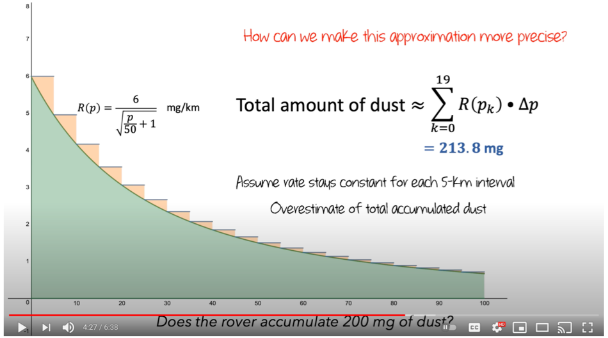
Total Accumulation is the Limit of These Riemann Sums = Definite Integral of Rate Function

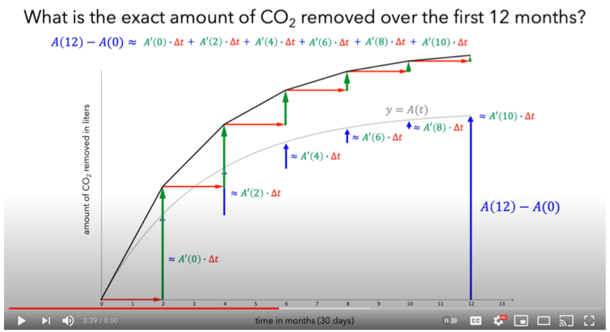


Riemann Sums to Fundamental Theorem

- Assume rate is constant over successive uniform intervals of the independent quantity's variation.
- Total accumulation is approximated by the sum of accumulations over these intervals.
- Smaller intervals over which rate is assumed constant can yield better approximations to total accumulation.
- The exact total accumulation is the change in the values of an antiderivative.







Our Ongoing Research Study

- Demographics
  - Two semesters (fall and spring)
  - 25 instructors providing data
  - 35+ instructors used the videos
  - 19 institutions
  - 817 first-semester calculus students
    - 276 students indicated they had NOT previously taken college calculus
    - 64 said yes
    - 477 did not answer

Our Ongoing Research Study

- Method
  - Basic Demographic Questions
  - Pre-Test & Posttest Questions per video
  - Gain Scores
  - Different treatments (briefly mention)
    - Intellectual Need (IN) Video, or IN Task, or Both
    - Outline
    - Control
  - Interviews (not this talk)
    - 4 video sets
    - Eye-tracking

Big Picture Results

- Students tended to do better on post-video questions than pre-video questions
- Students who saw video 0s did no differently than students who saw nothing (control)
- Students who saw the video outline did better than students who saw nothing (control)
- Students who saw the intellection need task did no differently than students who did not

Exploratory Results: Pre-Test/Posttest Gains by Video

WARNING: LEARING IS HARD

Results do not provide a “magical” solution.

Lowest: Why?

- Mismatch between video content and questions?
- Bad video? We hope not.
- Post video questions were too difficult?
- Etc.

- Results are mixed BUT…
- Students tended to do better on post-video questions than pre-video questions

Highest: Why?

- See list for lowest but replace "difficult" with easy. We argue this is not the case.
- More interesting content.
- More to learn.
- Etc.

Video Set	% gains
02_Constant Rate of Change	0.09410796443
03_Graphing Constant Rate of Change	2.758162701
04_[illegible]	[illegible]
05_Graphing Varying Rates of Change	-37.16455455
06_Average [illegible]	[illegible]
07_Approximating Instantaneous Rates of Change	-4.420676393
08_Continuity	16.86746633
09_Differentiability and Local Linearity	8.662020997
13_Limit Definition of Derivative	3.52595059
14_Using the Limit Definition of Derivative	15.65297126
15_Interpreting Derivatives	17.7244582
16_Secant Lines and Tangent Lines	3.560805715
17_Graphing Derivatives	-7.780228009
18_Basic Derivative Rules	6.4758283
19_Product Rule	21.22959151
20_Quotient Rule	35.08325361
21_Chain Rule	32.74714851
22_LHopitals Rule	4.220311123
23_Mean Value Theorem	23.06008474
24_Related Rates	25.24950305
25_Implicit Differentiation	24.03473078
26_Optimization Intro	8.221186972
27_[illegible]	[illegible]
28_Riemann Sums Introduction	59.81554315
29_[illegible]	[illegible]

Exploratory Results: Gains by Instructor

INSTRUTION MATTERS!!!

- Results vary by instructor.

Instructor	% gains
Instructor 1	11.8382241
Instructor 2	0.1417941176
Instructor 3	11.41734289
Instructor 4	11.76520975
Instructor 5	4.010497449
Instructor 6	-1.152676516
Instructor 7	11.35070596
Instructor 8	10.59812657
Instructor 9	8.416074799
Instructor 10	7.07258658
[illegible]	[illegible]
Instructor 12	15.71467571
Instructor 13	15.46328279
Instructor 14	18.78133096
[illegible]	[illegible]
Instructor 16	7.58287143
Instructor 17	12.07571955
Instructor 18	11.34287339
Instructor 19	14.45395824
Instructor 20	10.32209866
Instructor 21	10.30958626
Instructor 22	10.02749633
Instructor 23	7.415725806
Instructor 24	3.127320619
Instructor 25	12.53074494

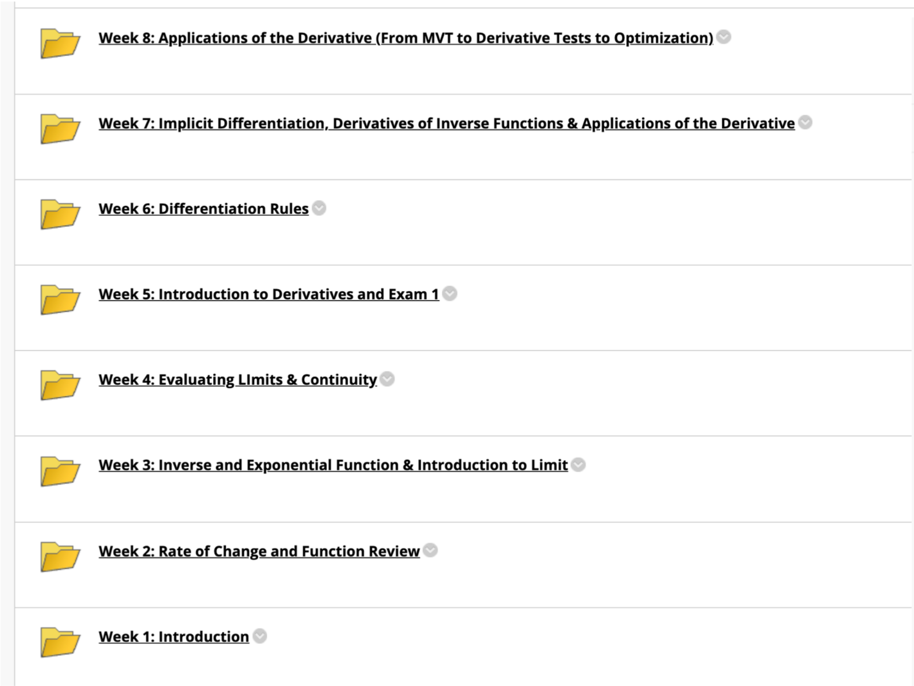
Highest Gains: Why?

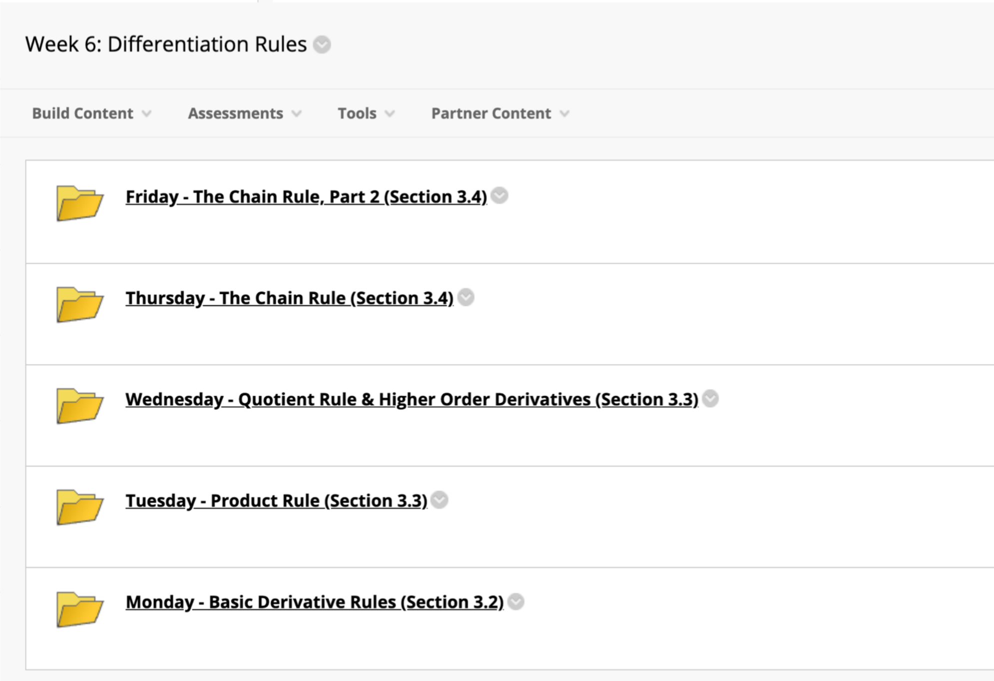
- Very involved instructors.
- Integrated videos into classroom (not merely homework assignments)
- Knowledgeable of QR

I am Instructor 13:
Why did my class have higher gains?

- Integrated into what I did during class (pre-COVID).
  - Videos were assigned prior to class and we discussed videos during class.
  - Videos were for a grade.
  - Content from videos might show up on quizzes and exams.

Higher gains during COVID? LMS Integration





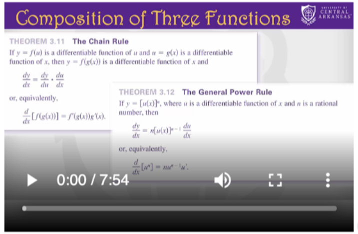
Higher gains during COVID? LMS Integration

Chain Rule

Enabled: Statistics Tracking

This module is designed to explain an efficient rule for taking the derivative of the compositi
explain how rate of change and function composition are related, provides opportunity to pr

Composition of Three Functions



- Download video file: 3.4 Composition of Three Functions.mp4

Supplementary Videos

Here are videos you are **not required** to watch but are encouraged to watch if needing or looking for addition

HW Assignment

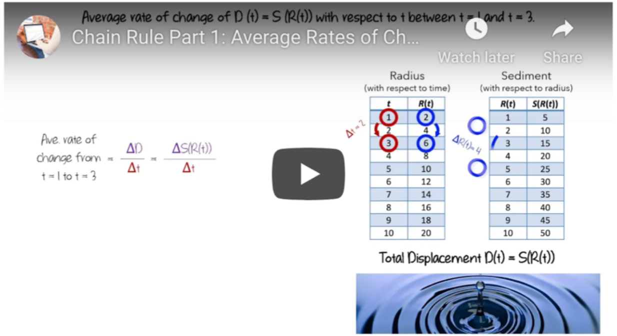
WebAssign HW 14 over the product rule and quotient rule is due tonight.

WebAssign HW 15 over the chain rule is due Monday night.

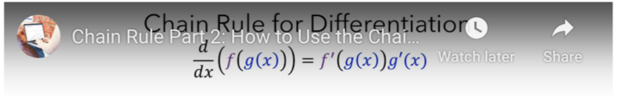
Chain Rule PreVideo Questions

PreVideo Questions are intended to get you thinking about the concept. You are not expected to know all answers pe
try. PreVideo Q uestions do NOT count for a grade.

Chain Rule Part 1: Average Rates of Change of Compositions of Functions



Chain Rule Part 2: How to Use the Chain Rule



Concluding Remarks

- Reasoning about quantities, including amounts of change, and how they co-vary can be leveraged to provide a reasonable explanation for calculus concepts and important theorems.
    - Know your audience. They are likely NOT mathematics majors. They are STEM majors who need to reason rigorously but this not mean they need formal proof.
- The video format provides alternative ways in which content can be presented dynamically. This allows for ”real-time” demonstrations of the variation inherent in most calculus problems.
- Videos can provide opportunity to create a more active classroom by moving some instruction outside of class BUT…
- How you integrate the videos into your classroom may be the single most important factor influencing student learning of the content presented in videos.
    - Merely assigning the videos, may result in no tangible learning gains.

THANK YOU

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